Optimizations and Extensions for Weighted CFG Parsers

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Outline

Pruning

Pruning for CKY parsing Pruning for deductive parsing

k-best parsing



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- During CKY or deductive parsing many items are explored which are not part of the best derivation
- Idea: avoid items that are not part of the best derivation to speed up parsing
- Problem: How can we know these items in advance?
- Practical solution: Use simple methods but take the risk of finding suboptimal derivation.

Consider this slightly modified (red) version of the CKY algorithm:

Require: weighted binary cfg (N, Σ, P, S, μ) , word $t_1 \dots t_n$ where $t_1, \dots, t_n \in \Sigma$ **Ensure:** family $(c_{i,i,A} \in \mathbb{R} \mid 0 \le i < j \le n, A \in N)$ such that, for all i, j, A, $c_{i,i,A} = \max\{\mu(d) \mid d \in D_G^A(t_{i+1} \dots t_i)\} \cup \{0\}$ 1: function $CKY(P, \mu, t_1 \dots t_n)$ $(c_{i,i,A} := 0 \mid 0 \le i \le j \le n, A \in N)$ 2: 3: for $1 \le i \le n$ do 4: for $A \rightarrow t_i \in P$ do 5: $c_{i-1,i,A} := \max\{c_{i-1,i,A}, \mu(A \to t_i)\}$ 6: for 2 < r < n do 7: for 0 < i < n - r do i := i + r8: for $m \in \{i + 1, i + 2, \dots, j - 1\}$ do 9: 10: for $B, C \in N$ do 11: for $A \in N$ such that $A \rightarrow BC \in R$ do $c_{i,i,A} := \max\{c_{i,i,A}, \mu(A \to BC) \cdot c_{i,m,B} \cdot c_{m,i,C}\}$ 12:

13: return c

An abstract "pruning operation" on the chart can be included (lines 6, 15). We speed up the algorithm by skipping the application of cfg rules to chart cells with 0 probability (line 12).

```
Require: weighted binary cfg (N, \Sigma, P, S, \mu), word t_1 \dots t_n where t_1, \dots, t_n \in \Sigma
Ensure: family (c_{i,i,A} \in \mathbb{R} \mid 0 \le i \le j \le n, A \in N) such that, for all i, j, A, c_{i,i,A} \le \max\{\mu(d) \mid d \in A\}
      D_{G}^{A}(t_{i+1} \dots t_{i}) \} \cup \{0\}
  1: function CKY(P, \mu, t_1 \dots t_n)
           (c_{i,i,A} := 0 \mid 0 \le i \le j \le n, A \in N)
  2:
  3:
          for 1 \le i \le n do
  4:
                for A \rightarrow t_i \in P do
  5:
                     c_{i-1,i,A} := \max\{c_{i-1,i,A}, \mu(A \to t_i)\}
  6:
                (c_{i-1,i,A} \mid A \in N) := \operatorname{prune}((c_{i-1,i,A} \mid A \in N))
 7:
           for 2 < r < n do
 8:
                for 0 < i < n - r do
 9:
                    i := i + r
                    for m \in \{i + 1, i + 2, \dots, j - 1\} do
10:
                          for B, C \in N do
11:
12:
                              if c_{i,m,B} = 0 or c_{m,i,C} = 0 then continue
                              for A \in N such that A \rightarrow BC \in R do
13:
                                   c_{i,i,A} := \max\{c_{i,i,A}, \mu(A \to BC) \cdot c_{i,m,B} \cdot c_{m,i,C}\}
14:
15:
                     (c_{i,i,A} \mid A \in N) := \operatorname{prune}((c_{i,i,A} \mid A \in N))
16:
           return c
```

How can the pruning operation be implemented?

- Threshold beam (set all cell probabilities to 0 if worse than θ · best cell probability) Require: family c = (c_{i,j,A} ∈ ℝ | A ∈ N), threshold θ ∈ [0,1] Ensure: family (c_{i,j,A} ∈ ℝ | A ∈ N)
 - 1: function PRUNE(c)
 - 2: $m = \max_{A \in N} \{ c_{i,j,A} \mid A \in N \}$
 - 3: for $A \in N$ do
 - 4: **if** $c_{i,j,A} < m \cdot \theta$ then
 - $c_{i,j,A} := 0$
 - 6: return c

5:

How can the pruning operation be implemented?

- **•** Threshold beam (set all cell probabilities to 0 if worse than θ · best cell probability) **Require:** family $c = (c_{i,i,A} \in \mathbb{R} \mid A \in N)$, threshold $\theta \in [0,1]$ **Ensure:** family $(c_{i,j,A} \in \mathbb{R} \mid A \in N)$
 - 1: function PRUNE(c)
 - 2: $m = \max_{A \in \mathcal{N}} \{ c_{i,j,A} \mid A \in \mathcal{N} \}$
 - 3: for $A \in N$ do
 - 4: if $c_{i,i,A} < m \cdot \theta$ then 5:

$$c_{i,j,A} := 0$$

- 6: return c
- Fixed-sized beam (set all but n best cell probabilities to 0)

```
Require: family c = (c_{i,i,A} \in \mathbb{R} \mid A \in N), size 1 \le n \le |N|
Ensure: family (c_{i,j,A} \in \mathbb{R} \mid A \in N)
```

- 1: function PRUNE(c)
- $[s_1,\ldots,s_n] = n\text{-best}\{c_{i,i,A} \mid A \in N\}$ 2:
- 3: for $A \in N$ do
- 4: if $c_{i,i,A} < s_n$ then
- 5: $c_{i,i,A} := 0$
- 6: return c

n-best(C) returns list of the *n* highest values in C in descending order

Pruning for CKY parsing- implementation considerations

- No changes to data structures required.
- More speed-ups might be obtained by not adding items to chart which for sure would later be pruned:
 - Threshold beam: store weight m of currently best item. If new item has weight m below $\theta \cdot m$, it is save to prune immediately.
 - Fixed-size beam: store weights of the n best items. If the weight of new is below of worst item, prune immediately.

Original algorithm for deductive parsing.

Require: weighted binary cfg (N, Σ, P, S, μ) , word $t_1 \dots t_n$ where $t_1, \dots, t_n \in \Sigma$ **Ensure:** family $(c_{i,i,A} : \mathbb{R} \mid 0 \le i < j \le n, A \in N)$ such that $c_{i,i,A} = \max\{\mu(d) \mid d \in D_C^A(t_{i+1} \dots t_i)\} \cup \{0\}$ 1: function DEDUCE($P, \mu, t_1 \dots t_n$) *queue* := { $(i - 1, A, i, \mu(A \rightarrow t_i)) | 1 \le i \le n, A \rightarrow t_i \in P$ } 2: $(c_{i,i,A} := 0 \mid 0 \le i \le j \le n, A \in N)$ 3. 4: while queue $\neq \emptyset$ do $(i, A, j, w) := \operatorname{argmax}_{(i, A, j, w) \in queue} w$ 5: queue $\setminus = \{(i, A, j, w)\}$ 6: 7: if $c_{i,i,A} = 0$ then 8: $C_i i A := W$ aueue $\cup = \{(i, A', i', \mu(A' \rightarrow AC) \cdot w \cdot c_{i,i',C}) \mid A' \rightarrow AC \in P\}$ 9: queue $\bigcup = \{(i', A', i, \mu(A' \rightarrow BA) \cdot c_{i', i, B} \cdot w) \mid A' \rightarrow BA \in P\}$ 10: queue $\cup = \{(i, A', j, \mu(A' \rightarrow A) \cdot w) \mid A' \rightarrow A \in P\}$ 11: 12: return c

Pruning operation on queue is added.

Require: weighted binary cfg (N, Σ, P, S, μ) , word $t_1 \dots t_n$ where $t_1, \dots, t_n \in \Sigma$ **Ensure:** family $(c_{i,i,A} : \mathbb{R} \mid 0 \le i < j \le n, A \in N)$ such that $c_{i,i,A} \leq \max\{\mu(d) \mid d \in D_{C}^{A}(t_{i+1} \dots t_{i})\} \cup \{0\}$ 1: function DEDUCE($P, \mu, t_1 \dots t_n$) *queue* := { $(i - 1, A, i, \mu(A \to t_i)) | 1 \le i \le n, A \to t_i \in P$ } 2: 3: $(c_{i,i,A} := 0 \mid 0 \le i \le j \le n, A \in N)$ 4: while queue $\neq \emptyset$ do 5: $(i, A, j, w) := \operatorname{argmax}_{(i, A, i, w) \in queue} w$ 6: queue $\setminus = \{(i, A, j, w)\}$ 7: if $c_{i,i,A} = 0$ then 8: $C_{i,i,A} := W$ queue $\cup = \{(i, A', j', \mu(A' \rightarrow AC) \cdot w \cdot c_{i,j',C}) \mid A' \rightarrow AC \in P\}$ 9: queue $\cup = \{(i', A', j, \mu(A' \rightarrow BA) \cdot c_{i', j, B} \cdot w) \mid A' \rightarrow BA \in P\}$ 10: queue $\cup = \{(i, A', i, \mu(A' \rightarrow A) \cdot w) \mid A' \rightarrow A \in P\}$ 11: 12: prune(*queue*)

13: return c

Again two options for pruning:

Threshold beam (remove each queue item if its probability is worse than $\theta \cdot \text{prob.}$ of best queue item)

 $\begin{array}{ll} \mbox{Require: set } queue \subseteq \mathbb{N} \times \textit{N} \times \mathbb{N} \times \mathbb{R}, \mbox{ threshold } \theta \in [0,1] \\ \mbox{Ensure: set } queue' \subseteq \mathbb{N} \times \textit{N} \times \mathbb{N} \times \mathbb{R} \end{array}$

1: function PRUNE(queue)

2:
$$m = \max_{(i,A,j,w) \in queue} w$$

3: return
$$\{(i, A, j, w) \in queue \mid w > \theta \cdot m\}$$

Again two options for pruning:

• Threshold beam (remove each queue item if its probability is worse than $\theta \cdot \text{prob.}$ of best queue item)

```
Require: set queue \subseteq \mathbb{N} \times N \times \mathbb{N} \times \mathbb{R}, threshold \theta \in [0, 1]
Ensure: set queue' \subseteq \mathbb{N} \times N \times \mathbb{N} \times \mathbb{R}
```

- 1: function PRUNE(queue)
- 2: $m = \max_{(i,A,j,w) \in queue} w$
- 3: return $\{(i, A, j, w) \in queue \mid w > \theta \cdot m\}$

 Fixed-sized beam (only keep the n most probable queue items) Require: set queue ⊆ N × N × R, size n ∈ N Ensure: set queue' ⊆ N × N × N × R

1: function PRUNE(queue)

2:
$$[i_1, \ldots, i_n] = n$$
-best(queue) w.r.t. 4th tuple component

3: return $\{i_1, ..., i_n\}$

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- Beware: Items for large spans are often more probable than items for small spans. Risk of pruning "good" large items in favour of "bad" small items. (Solution: see A*-star parsing below)

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k-best parsing

▶ Problem: syntactic ambiguity, e.g., "She saw the astronomer with the telescope."

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- Solution: return multiple parse trees per sentence.
- ▶ Goal: given a sentence w, a PCFG G, and a positive integer k, find the k most probable derivations of G for w

k-best parsing – naïve algorithm

Similar to CKY algorithm. Uses functions: $\operatorname{sort}(c)$ – sorts a set c of tuples (according to 2nd component) $\operatorname{take}(k, \ell)$ – returns first k elements of list ℓ

Require: $k \in \mathbb{N}$, weighted binary CFG (N, Σ, S, R, μ) , word $t_1 \cdots t_n$ **Ensure:** k most probable parse trees of PCFG for $t_1 \cdots t_n$

1: function KBEST(k, R,
$$\mu$$
, $t_1, ..., t_n$)
2: $b[i, j, A] := []$ for each cell (i, j, A)
3: for $i \in \{0, ..., n-1\}$ and $A \in N$ do
4: $c := \{(A(t_{i+1}), w) \mid A \to t_{i+1} \text{ in } R, w = \mu(A \to t_{i+1})\}$
5: $b[i, i+1, A] = \text{take}(k, \text{sort}(c))$
6: for $z \in \{2, ..., n\}$ do
7: for $i \in \{0, ..., n-z\}$ do
8: $j := i + z$
9: for $A \in N$ do
10: $c := \{(A(d_1, d_2), w) \mid A \to BC \text{ in } R, m \in \{i + 1, ..., j - 1\}, (d_1, w_1) \in b[i, m, B], (d_2, w_2) \in b[m, j, C], w = \mu(A \to BC) \cdot w_1 \cdot w_2\}$
11: $b[i, j, A] = \text{take}(k, \text{sort}(c))$

12: return *b*[0, *n*, *S*]

k-best parsing – implementation of merging

10:
$$c := \{ (A(d_1, d_2), w) \mid A \to BC, m \in \{i + 1, \dots, j - 1\}, (d_1, w_1) \in b[i, m, B], (d_2, w_2) \in b[m, j, C], w = \mu(A \to BC) \cdot w_1 \cdot w_2 \}$$

11: $b[i, j, A] = take(k, sort(c))$

can be implemented as

10:
$$b[i, j, A] := []$$

11: for $m \in \{i + 1, ..., j - 1\}$ do
12: for $A \to BC$ in R do
13: $c := \{(A(d_1, d_2), w) \mid (d_1, w_1) \in b[i, m, B], (d_2, w_2) \in b[m, j, C], w = \mu(A \to BC) \cdot w_1 \cdot w_2\}$
14: $b[i, j, A] := mergeAndTakeK(k, b[i, j, A], c)$

here mergeAndTakeK (k, ℓ, c) returns the list of k-best items in the union of list ℓ and set c

k-best parsing - merging more efficiently [HC05]

the set *c* can be computed lazily

avoid considering all k^2 items obtained by combining each of b[i, m, B] with b[m, j, C]instead, we only combine the best items of b[i, m, B] with the best items of b[m, j, C]: if the combination of the u-th best of b[i, m, B] and v-th best of b[m, j, C] was considered, then consider also the combinations (u + 1, v) and (u, v + 1)

2			2				2			
2			2	3	⇒		2	3	4	
$0 \mid [1] \Rightarrow$	÷		0	1	[2]	⇒	0	1	2	(4)
1 2	2 4	Г		1	2	4		1	2	4
(a)				(b)			6	:)	

2			
2	3	4	
0	1	2	(4)
	1	2	4

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 - ▶ Why?: Usually items with small spans are more probable than items with large spans.
 - This is counteracted by future costs which are higher for items with small spans.
- ▶ Klein and Manning [KM03] propose several admissible heuristics.
- A heuristic may also be useful when pruning items during CKY parsing.

A*-parsing – Viterbi outside score

We use the admissible heuristic $\operatorname{out:}$

$$\operatorname{out}(A) = \max_{d \in D_G, u, w \in \Sigma^* : S \stackrel{d}{\Rightarrow}_G uAw} \operatorname{weight}(d)$$

It can be computed by a variant of the inside/outside algorithm:

1: function Inside 2: for $A \in N$ do 3: $in(A) := \max(\{\mu(A \to \alpha) \mid A \to \alpha \in R\} \cup \{0\})$ 4: while not converged do 5: for $A \in N$ do 6: $in(A) = \max(\{in(A)\} \cup \{\mu(A \to BC) \cdot in(B) \cdot in(C) \mid A \to BC \text{ in } R\}$ $\cup \{\mu(A \to B) \cdot in(B) \mid A \to B \text{ in } R\})$ 7: function OUTSIDE set $out(B) := \begin{cases} 1 & B = S \\ 0 & otherwise \end{cases}$ for each $B \in N$ 8: 9: while not converged do 10: for $B \in N$ do 11: $\operatorname{out}(B) := \max(\{\operatorname{out}(B)\} \cup \{\operatorname{out}(A) \cdot \mu(A \to BC) \cdot \operatorname{in}(C) \mid A \to BC \text{ in } R\}$ $\cup \{ \operatorname{out}(A) : \mu(A \to CB) : \operatorname{in}(C) \mid A \to CB \text{ in } R \}$ $\cup \{ \operatorname{out}(A) \cdot \mu(A \to B) \mid A \to B \text{ in } R \} \}$

A*-parsing – parsing algorithm with heuristic

The out-value is now simply included when selecting the best item from the queue:

```
Require: weighted binary cfg (N, \Sigma, P, S, \mu), word t_1 \dots t_n where t_1, \dots, t_n \in \Sigma
Ensure: family (c_{i,i,A} : \mathbb{R} \mid 0 \le i < j \le n, A \in N) such that
       c_{i,i,A} = \max\{\mu(d) \mid d \in D_G^A(t_{i+1} \dots t_i)\} \cup \{0\}
 1: function DEDUCE(P, \mu, t_1 \dots t_n)
 2:
            queue := {(i - 1, A, i, \mu(A \to t_i)) | 1 < i < n, A \to t_i \in P}
           (c_{i i A} := 0 \mid 0 \le i \le i \le n, A \in N)
 3:
 4:
           while queue \neq \emptyset do
 5:
                 (i, A, j, w) := \operatorname{argmax}_{(i, A, i, w) \in gueue} w \cdot \operatorname{out}(A)
 6:
                 queue \setminus = \{(i, A, j, w)\}
 7:
                 if c_{i,i,A} = 0 then
 8:
                      C_{i,i,A} := W
                      queue \cup = \{(i, A', j', \mu(A' \rightarrow AC) \cdot w \cdot c_{i,j',C}) \mid A' \rightarrow AC \in P\}
 9:
                      queue \cup = \{(i', A', j, \mu(A' \rightarrow BA) \cdot c_{i', j, B} \cdot w) \mid A' \rightarrow BA \in P\}
10:
                      queue \cup = \{(i, A', j, \mu(A' \rightarrow A) \cdot w) \mid A' \rightarrow A \in P\}
11:
12:
            return c
```

Pruning for deductive parsing - in the context of CKY

```
Can A*-parsing be applied for CKY parsing?
Yes: in combination with pruning:
```

Threshold beam

```
Require: family c = (c_{i,j,A} \in \mathbb{R} \mid A \in N), threshold \theta \in [0,1]
Ensure: family (c_{i,j,A} \in \mathbb{R} \mid A \in N)
```

1: function PRUNE(c)

2:
$$m = \max_{A \in N} \{ c_{i,j,A} \cdot \operatorname{out}(A) \mid A \in N \}$$

3: for
$$A \in N$$
 do

4: **if**
$$c_{i,j,A} \cdot \text{out}(A) < m \cdot \theta$$
 then

$$5: \qquad c_{i,j,A} := 0$$

```
6: return c
```

Pruning for deductive parsing – in the context of CKY

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Can A*-parsing be applied for CKY parsing?
Yes: in combination with pruning:
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Threshold beam
     Require: family c = (c_{i,i,A} \in \mathbb{R} \mid A \in N), threshold \theta \in [0,1]
     Ensure: family (c_{i,j,A} \in \mathbb{R} \mid A \in N)
       1: function PRUNE(c)
       2:
                m = \max_{A \in \mathbb{N}} \{ c_{i,j,A} \cdot \operatorname{out}(A) \mid A \in \mathbb{N} \}
       3:
                for A \in N do
       4:
                     if c_{i,i,A} \cdot \text{out}(A) < m \cdot \theta then
       5:
                          c_{i,i,A} := 0
       6:
                return c
Fixed-size beam
     Require: family c = (c_{i,i,A} \in \mathbb{R} \mid A \in N), size 1 \le n \le |N|
    Ensure: family (c_{i,j,A} \in \mathbb{R} \mid A \in N)
       1: function PRUNE(c)
                [s_1,\ldots,s_n] = n\text{-best}\{c_{i,j,A} \cdot \text{out}(A) \mid A \in N\}
       2:
       3:
                for A \in N do
```

```
if c_{i,i,A} \cdot \text{out}(A) < s_n then
4:
5:
```

```
c_{i,i,A} := 0
```

6: return c

- [Atk+86] M. D. Atkinson et al. "Min-max Heaps and Generalized Priority Queues". In: Commun. ACM 29.10 (Oct. 1986), pp. 996–1000. ISSN: 0001-0782. DOI: 10.1145/6617.6621. URL: http://doi.acm.org/10.1145/6617.6621.
- [HC05] Liang Huang and David Chiang. "Better k-best parsing". In: Proceedings of the Ninth International Workshop on Parsing Technology. Association for Computational Linguistics. 2005, pp. 53–64.
- [KM03] Dan Klein and Christopher D. Manning. "A* Parsing: Fast Exact Viterbi Parse Selection". In: Proceedings of the 2003 Human Language Technology Conference of the North American Chapter of the Association for Computational Linguistics. 2003, pp. 119–126. URL: https://www.aclweb.org/anthology/N03-1016.