Formale Baumsprachen

Task 18 (monadic second-order logic II)

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet and $\mathcal{V} = \{x, y, U\}$. Construct MSO-formulas φ_1, φ_2, and φ_3 such that $\mathrm{Fr}(\varphi_1), \mathrm{Fr}(\varphi_2), \mathrm{Fr}(\varphi_3) \subseteq \mathcal{V}$ and for every $\xi \in T_\Sigma$ and $\mathcal{V}$-assignment ρ for ξ:

(a) $(\xi, \rho) \models \varphi_1$ iff there is a downward path from the node $\rho(x)$ to the node $\rho(y)$ in ξ, i.e. there is a $w \in \mathbb{N}^*$ such that $\rho(y) = \rho(x)w$.

(b) $(\xi, \rho) \models \varphi_2$ iff $\rho(U)$ is the set of all positions w in ξ such that $\xi|_w = \sigma(\alpha, \beta)$.

(c) $(\xi, \rho) \models \varphi_3$ iff for every node in ξ labeled by σ, none of its child nodes is labeled by γ.

Task 19 ($\mathrm{Rec} \subseteq$ *MSO-definable)*

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be a ranked alphabet. Consider the bottom-up deterministic fta $\mathcal{A} = (Q, \Sigma, \delta, F)$ with $Q = \{0, 1\}$, $F = \{1\}$, and $\delta_\sigma(q_1, q_2) = \max\{q_1, q_2\}$ for every $q_1, q_2 \in Q$, $\delta_\gamma(q) = 1$ for every $q \in Q$, and $\delta_\alpha() = 0$. Use the construction from the lecture to show that $L(\mathcal{A})$ is MSO-definable.

Task 20 (MSO-definable $\subseteq \mathrm{Rec}$*)*

(a) Recall the following Lemma from the lecture:

Lemma. Let Σ be a ranked alphabet and $\mathcal{V} \subseteq_{\mathrm{fin}} \mathcal{V}_1$. Then $T^{\mathrm{v}}_{\Sigma_\mathcal{V}}$ is recognizable.

In the proof we required a family of languages $(L_x \mid x \in \mathcal{V})$ where for every $x \in \mathcal{V}$:

$$L_x = \{\xi \in T_{\Sigma_\mathcal{V}} \mid x \text{ occurs exactly once in } \xi\}.$$

Construct an automaton $\mathcal{A}_x$ for every $x \in \mathcal{V}$ such that $L(\mathcal{A}_x) = L_x$.

(b) Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ and $\varphi = \exists x. \mathrm{label}_\gamma(x)$. Use the construction from the lecture to show that $L(\varphi)$ is recognizable.

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