Formale Übersetzungsmodelle

Exercise 23 (Property (T2))

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ and $\Delta = \Sigma \cup \{\gamma'^{(1)}\}$ be r.a. Consider the td-tt $T = \big(Q, \Sigma, \Delta, \{q_0\}, R\big)$, where $Q = \{q_0, q_1\}$, and R contains the rules

$$\begin{aligned} p(\sigma(x_1, x_2)) &\to \sigma(p(x_1), p(x_2)) \\ p(\sigma(x_1, x_2)) &\to \gamma'(q_1(x_1)) \\ p(\sigma(x_1, x_2)) &\to \gamma'(q_1(x_2)) \\ p(\gamma(x_1)) &\to \gamma(q_0(x_1)) \\ q_0(\alpha) &\to \alpha \end{aligned}$$

for every $p \in Q$.

(a) Describe the tree transformation computed by T. How does it relate to the property (T2)?

(b) Can you give a bu-tt B with $\tau(B) = \tau(T)$?

Exercise 24 (Decomposition of TOP)

Let $\Sigma = \big\{\gamma^{(1)}, \alpha^{(0)}\big\}$ and $\Delta = \big\{\sigma^{(2)}, O^{(1)}, E^{(1)}, \alpha^{(0)}\big\}$ be r.a., and let $T = \big(\{q_E, q_O\}, \Sigma, \Delta, \{q_O\}, R\big)$ be a td-tt, where R contains the rules

$$\begin{aligned} q_E(\alpha) &\to E(\alpha) \\ q_E(\gamma(x_1)) &\to E(\sigma(q_O(x_1), q_O(x_1))) \\ q_O(\alpha) &\to O(\alpha) \\ q_O(\gamma(x_1)) &\to O(\sigma(q_E(x_1), q_E(x_1))) \end{aligned}$$

Apply the method from the lecture and construct a tree homomorphism (i.e. a homomorphism td-tt) H and a linear td-tt T' such that $\tau(H) \circ \tau(T') = \tau(T)$.

Exercise 25 (l-TOP ⊆ l-BOT)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$ be r.a. Consider the linear td-tt $T = (\{q, p\}, \Sigma, \Sigma, \{q\}, R)$ with rules in R as follows:

$$\begin{aligned} q(\sigma(x_1, x_2)) &\to \sigma(p(x_1), \alpha) \\ p(\sigma(x_1, x_2)) &\to \sigma(\alpha, q(x_2)) \\ q(\alpha) &\to \alpha \\ p(\alpha) &\to \alpha \end{aligned}$$

(a) Describe $\tau(T)$.

(b) Construct, as described in the lecture, a linear bu-tt B such that $\tau(B) = \tau(T)$.

Exercise 26 (Decomposition of TOP (ii))

Let $\Sigma = \{X^0, *^{(2)}, +^{(2)}\} \cup \{0^{(0)}, \ldots, 5^{(0)}\}$ be a r.a., and let $T = (\{d, e\}, \Sigma, \Sigma, \{d\}, R)$ be a td-tt, where R contains the rules

$$
\begin{aligned}
d(i) &\to 0 \qquad d(X) \to 1 \\
e(i) &\to i \qquad e(X) \to X \\
d(*(x_1, x_2)) &\to +\big(*(d(x_1), e(x_2)), *(e(x_1), d(x_2))\big) \\
d(+(x_1, x_2)) &\to +(d(x_1), d(x_2)) \\
e(+(x_1, x_2)) &\to +(e(x_1), e(x_2)) \\
e(*(x_1, x_2)) &\to *(e(x_1), e(x_2))
\end{aligned}
$$

for every $0 \leq i \leq 5$. Apply the method from the lecture and construct a tree homomorphism (i.e. a homomorphism td-tt) H and a linear td-tt T' such that $\tau(H) \circ \tau(T') = \tau(T)$.